Now, using a well-known addition theorem for the theta functions,

$$x\sqrt{1-x^2} = j\frac{K'H(M)H_1(M)H(u)H_1(u)}{K\theta^2(0)H(M+u)H(M-u)} \quad (11)$$

so that

$$P_N(x)/(x\sqrt{1-x^2}) = C\frac{H^N(M+u) - H^N(M-u)}{H(u)H_1(u)H^{n-1}(M+u)H^{n-1}(M-u)} \quad (12)$$

where C is a real constant. Denoting the fraction on the right by $F(u)$, the argument will be complete when we show that the function, $f(x)$, defined by $F(u)$ together with (6), is an even polynomial in x with real coefficients.

That $f(x)$ is a polynomial of degree $N-2$ is evident from the following.

1) $f(x)$ is single valued.

2) $f(x)$ is analytic everywhere except at the point of infinity.

3) The singularity of $f(x)$ at infinity is a pole of order $N-2$, for it is well known that an analytic function whose only singularity is a pole at infinity of order m is a polynomial of degree m. Then, to show that it is an even polynomial with real coefficients, it is sufficient to show that it is even and real on the portion of the real axis between $-x_1$ and $+x_1$, by the principle of analytic continuation.

Now $f(x)$ is single valued because $F(u)$ is doubly periodic in u with the same periodicity rectangle that x has as a function of u. So that, although to each value of x there corresponds an infinite number of values of u, each in turn gives the same value of $F(u)$. To show that $F(u)$ has the required property is a formal matter; and the reader is referred to [1]. One simply replaces u by $u+2K$ and $u+2jK'$, in turn, in the defining equations and shows that

$$F(u) = F(u+2K) = F(u+2jK') \quad (13)$$

making use of the fact that $NM=K$.

By the function of a function theorem for analytic functions, $f(x)$ is analytic except at the singularities of $F(u)$ as a function of u and the singularities of u as function of x. Both of these sets of singularities are readily seen to be finite in number. It follows then, from a well-known theorem for single-valued analytic functions with a finite number of singularities, that $f(x)$ cannot be bounded in the neighborhood of any of its singularities. Hence, in searching for the singularities of $f(x)$, we need not concern ourselves with the critical points of (6) since $f(x)$ can be unbounded only when $F(u)$ is unbounded. Moreover, because of the periodicity of $F(u)$, we may limit the search for singular points to values of u in a periodicity rectangle determined by $(\pm K, \pm jK')$. Now, since $H(u)$ is bounded in the finite plane, infinities of $F(u)$ occur only at zeros of its denominator. The zeros of $H(u)$ and $H_1(u)$ are simple and occur at $u=0$ and $u=-K$, but it is readily seen that they are cancelled by zeros of the numerator of $F(u)$. Thus the only singularities of $F(u)$ in the periodicity rectangle occur at $u=\pm M$. Both of these values correspond to $x=\infty$ and we conclude that $f(x)$ is analytic in the entire plane except at the point of infinity.

It is a formal matter to show that $f(x)/x^{N-2}$ approaches a finite limit as $x\to\infty$. Thus the only singularity of $f(x)$ is a pole of order $N-2$ at infinity.

Finally, from the mapping of the x plane (given in Fig. 1) and the evaluation of $P_N(x)$ as $\mathrm{Im}[y^n]$ for values of x between $-x_1$ and $+x_1$, it follows the $P_N(x)$ is a real odd function of x on this line segment. Thus $f(x)$ is a real even polynomial for values of x on this line segment.

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Computer Program Descriptions

Computer Solution of Transient and Time Domain Thin-Wire Antenna Problems

PURPOSE: SWIRE is a general purpose computer program which analyzes the transient and time domain electromagnetic behavior of straight-wire scatterers and antennas (both transmitting and receiving).

LANGUAGE: FORTRAN.

AUTHORS: Alethia M. Auckenthaler and C. Leonard Bennett, Sperry Rand Research Center, Sudbury, Mass. 01776.

AVAILABILITY: ASIS-NAPS Document No. NAPS-01541.

DESCRIPTION: A general purpose computer program that analyzes the transient and time domain electromagnetic behavior of transmitting, receiving, and scattering straight-wire antennas is presented. The program allows an arbitrary number of transmit or receive points, each with arbitrary source or load resistances and arbitrary distributed resistive loading along the wire length. The program also permits the computation of the far zone normalized field in arbitrary directions. The flexibility in both the input and output of this program and its applicability to the general time varying case allows the solution of a wide range of practical engineering problems.

The straight-wire scattering and antenna problem which is illustrated in Fig. 1 consists of a straight wire located on the x axis with some excitation. For the case of the scattering or receiving antenna problem the excitation is the x component of the incident wave E^i which makes an angle of θ^i with the plane perpendicular to the x axis. For the case of the transmitting antenna problem the excitation is a voltage generator with a source resistance R_g. These excitations produce currents $I(x)$ along the wire which in turn produce a far zone field H^s in the Ψ^s direction.

The technique used to solve this wire scattering problem [1], [2] is a specialization of the integral equation technique used in the time domain solution of the more general problem of scattering by surfaces [3]. Since the wire is assumed to be thin (e.g., the wire radius is much less than the width of an incident Gaussian shaped pulse), the wire current flows only in the axial direction and the more complicated surface integral equation reduces to a single space time scalar

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For program listing, order document NAPS-01541 from ASIS National Auxiliary Publications Service, c/o CCM Information Corporation, 909 Third Avenue, New York, N. Y. 10022; remitting $2.00 per microfiche or $5.00 per photocopy.

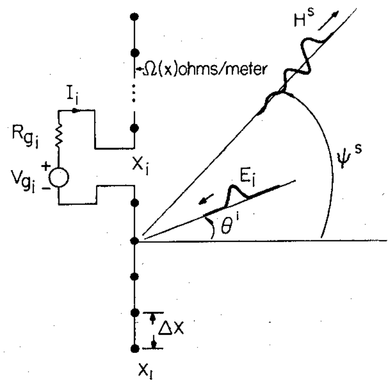

Fig. 1. Geometry of thin-wire antenna as scatterer, receiver, or radiator.

integro-differential equation for the wire currents. The form of this equation is similar to a one-dimensional wave equation which is then solved numerically on a digital computer by marching on in time. Once the currents have been found the far zone fields they produce are also computed numerically on the digital computer.

The input to the computer program is read in six data groups which control

1) type of problem to be analyzed;
2) wire geometry;
3) excitation;
4) wire loading;
5) far zone field computation;
6) output data options.

Instructions for preparing the data are presented to the user in the comment cards at the beginning of the main program. In addition, all input data are checked for maximum dimension allowances and reset if necessary. The units of time are light meters (one light meter is the time it takes a wave moving at the velocity of light to travel 1 m).

The incident plane wave for the case of the scattering or receive antenna problem is a Gaussian shaped pulse:

$$E^i(x, t) = -\sqrt{\frac{\mu}{\epsilon}} \frac{a_n}{\sqrt{\pi}} \exp\left[-a_n^2(t + x \sin\theta^i)^2\right]. \quad (1)$$

Note that the peak of this pulse reaches the origin at $t=0$. The generator voltage used for the case of the radiating antenna is a smoothed step waveform given by

$$V_g(t) = \int_{-\infty}^{t} E^i(0, t')dt'. \quad (2)$$

The width of the pulse in (1) or the rise time of the step in (2) is approximately $4/a_n$ light meters.

The program prints out the input data, the wire currents that are computed at each sample point in space time, and the far zone magnetic field normalized by the distance from the origin at each point in direction time.

The program has been run on both Univac 1108 and IBM 360 computers and requires approximately 43 000 words. Execution time on a Univac 1108 is found to be approximately

$$kN_wN_T(N_w + N_p)$$

where

- N_w number of wire sample points (NXW);
- N_p number of far field directions (NP);
- N_T number of time sample points;
- k 1.6×10^{-4} s.

Good agreement is found when the program results are compared with both experimental measurements and results computed by taking the inverse Fourier transform of frequency domain solutions.

This program computes the smoothed impulse response or the smoothed step response of these scattering and antenna problems. It should be pointed out that the response due to any time varying waveform can be computed from the impulse response by a simple convolution operation. In particular, the Fourier transform of the impulse response yields the entire frequency response directly. Thus a single time domain calculation for the impulse response solves a particular scattering or antenna problem for all excitations.

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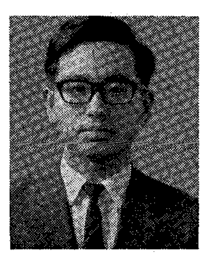

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